The Temperature Jump at the Inlet of the Continuous Phase in a Spray Column Heat Exchanger

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A spray column is characterized by a sharp drop in the driving force at the inlet of the continuous phase, the so-called "end effect." An earlier paper (*2*) reported the temperature jump at the inlet of the continuous phase, water, in a 7.5 cm. in diameter spray column heat exchanger, with a dispersed packing of kerosene drops as function of the holdup and the ratio of flow rates of the two phases. This work was extended to wider ranges of holdup and ratio of flow rates of the two phases, in a 15 cm. in diameter spray column. The column was operated with a dispersed packing of kerosene drops in water and with a temperature difference of 10°C. between inlets, at a mean temperature of 35°C. for cooling or heating drops and of 65°C., for cooling drops. Further details of the experimental procedure were given earlier (*4*). Figure 1 is a plot of the relative temperature jump as a function of the average holdup of drops and of the ratio of flow rates of the two phases, at an average temperature of 35°C. for cooling drops. Similar plots at the other operating conditions did not show any significant effect of temperature or direction of heat transfer. The only divergence from the earlier work (*2*) is that the minimum jump ratios were obtained at a holdup range of 0.20 to 0.40 instead of 0.24.

In the earlier work (*2*) and in Figure 1, the different curves of the temperature jump ratio appear to converge at zero holdup. This was earlier (*2*) assumed to be caused by longitudinal dispersion, due to the radial velocity profile of the water. The mechanism of heat transfer in a spray column is now better understood (*4*) and a meaningful qualitative and quantitative analysis of the temperature jump can now be made.

A short version of the physical model of the mechanism of heat transfer in a spray column (*4*) follows: Heat is transferred from a perfectly mixed drop to its perfectly mixed wake as it forms at the lower part of the column. In the major part of the column vibrations of the drop and collisions with adjacent drops cause shedding of elements of the wake, which are replaced by the continuous phase. The shed wake elements mix with the continuous phase. A highly mixed zone exists at the inlet of the continuous phase to the column and all streams leaving that zone are at the same temperature.

The explanation of the convergence of the temperature jump ratios at zero holdup is: As the holdup is decreased, the rate of collisions of drops decreases and the rate of wake shedding decreases. The limit at zero holdup corresponds to the flow of single drops, which give up their wakes only upon coalescing at the top of the column. At zero holdup almost all the heat lost by the drop is localized in its wake, which is totally transferred to the continuous phase only at the top of the column.

The existence of a minimum of the temperature jump ratio as function of holdup is caused by a combination of three factors which are affected differently by increased holdup.

1. The lengths of the mixing zone and the wake formation zone decrease and the length of the wake shedding zone increases with increased holdup (*4*). Therefore, more heat is transferred in the wake shedding zone with increased holdup.
2. The volume of wake elements shed per unit length of travel increases as the holdup increases from zero to 0.20, does not change significantly in the holdup range of 0.20 to 0.40 and decreases at higher holdups (*4*). Increased rate of wake shedding increases the rate of heat transfer in the wake shedding zone.
3. At holdups above 0.4, back mixing of drops and coalescence of drops become significant, and both increase with increased holdup (*3*). Therefore the rate of heat transfer within the column decreases with increased holdup at high holdups.

The larger conical entry section in the larger column retards the start of flooding with its attendant severe

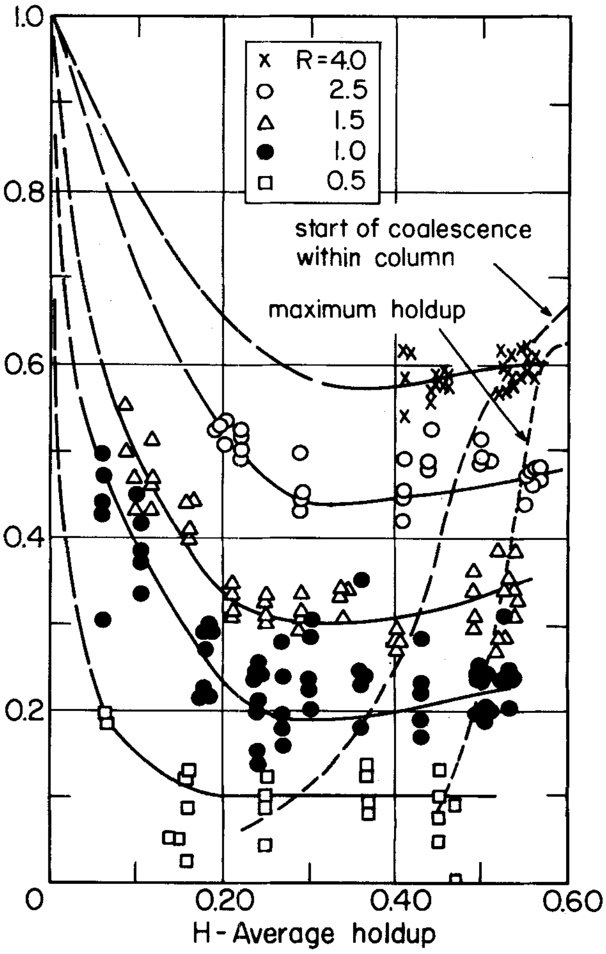

Fig. 1. The relative temperature jump at the water inlet as function of the average holdup and ratio of flow rates, at an average temperature of 35°C. for cooling kerosene drops.

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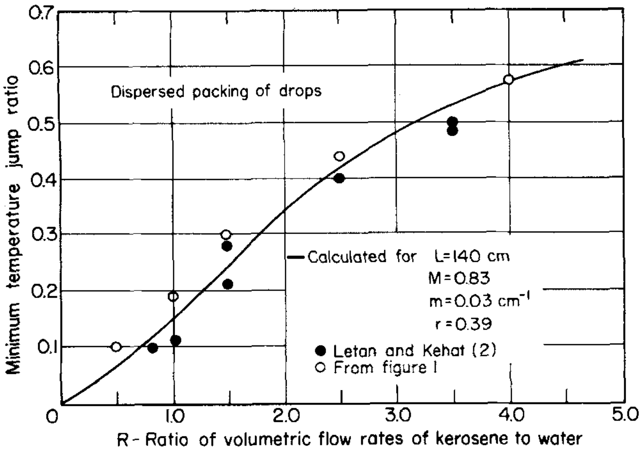

Fig. 2. Minimum temperature jump ratios as function of the ratio of the volumetric flow rates of kerosene to water.

coalescence of drops within the column and increased back mixing of drops (*3*). Therefore, in the larger column the increase in the temperature jump ratio begins at higher holdups than in the smaller column.

A quantitative analysis of the temperature jump can be made by the use of the mathematical expressions developed for the theoretical model (*4*). The temperature jump ratio is defined quantitatively by Equation (32) of (*4*) in the form:

$$\theta_a = 1 - \frac{1}{Rr\left(1 - \frac{1}{F}\right)} \qquad (1)$$

where F is the dimensionless temperature of the continuous phase at the top of the wake shedding zone, and is defined by equation (33) of (*4*). F is a function of M, m, L, r, and R. In the range of holdup of 0.2 to 0.4 and for the average drop size of approximately 3.5 mm. in this work, the values of M and m are approximately constant with $M = 0.83$ and $m = 0.03$ cm.$^{-1}$. At 35°C., $r = 0.39$. An average value of $L = 140$ cm. was assumed for the length of the wake shedding zone. The above equation, with the definition of F and the above values of the parameters, was used to calculate the temperature jump ratio as a function of the ratio of flow rates of the two phases; the theoretical curve is plotted in Figure 2. The experimental points of the minimum values of the temperature jump curves from Figure 1 and from Figure 5 of (*2*) are also shown. The agreement of the experimental data with the theoretical curve is good, thus substantiating the role of the wakes of the drops in the mechanism of heat transfer in spray column heat exchangers.

AXIAL DISPERSION AND THE PROPOSED MODEL

The end effects in an extraction or heat transfer spray column have commonly been attributed to axial dispersion of the two phases in the spray column. Sleicher (*8*), Miyauchi and Vermeulen (*7*), and Wilburn (*9*) have discussed the axial dispersion models theoretically. Hazelbeck and Geankoplis (*1*), and Mixon, et al. (*6*) have determined experimentally dispersion coefficients in spray columns. Temperature profiles, calculated from the theoretical models cannot account for the temperature profiles in the wake formation zone (*4*). Therefore, the axial dispersion models do not have general applicability to spray columns.

The wake shedding zone, however, consists of a series of mixing zones of the shed elements of wakes, similar to the well mixed vessel cascade model which is equivalent to the axial dispersion model. This similarity can also be shown quantitatively.

If the wake temperature in the wake shedding zone is eliminated from the heat balances for the continuous phase and the wakes [equations (11) and (13) of (*4*)] and the distance along the wake shedding zone is normalized by the use of $x = z/L$, the resultant equation is

$$\frac{(MR+1)M}{Lm}\,\frac{d^2t_c}{dx^2} + \frac{dt_c}{dx} + LRm(t_d - t_c) = 0 \qquad (2)$$

Similarly, from the heat balance for the dispersed phase [equation (8) of (*4*)] we obtain

$$\frac{dt_d}{dx} + \frac{mL}{r}\,(t_d - t_c) = 0 \qquad (3)$$

The comparable equations in axial dispersion terminology are

$$\frac{1}{N_{Pe_c}}\,\frac{d^2t_c}{dx^2} + \frac{dt_c}{dx} + N_c(t_d - t_c) = 0 \qquad (4)$$

$$\frac{1}{N_{Pe_d}}\,\frac{d^2t_d}{dx^2} - \frac{dt_d}{dx} - N_d(t_d - t_c) = 0 \qquad (5)$$

A comparison of Equations (2) and (4), and Equations (3) and (5) shows that the two models are equivalent in the wake shedding zone, provided that the axial dispersion coefficient is taken to be zero for the dispersed phase and if corresponding coefficients are identified.

We (*5*) have recently shown that for a dispersed packing of drops and a wide range of holdups, the drops flow in essentially plug flow, which upholds the assumption of an axial dispersion coefficient of zero for the dispersed phase. This, however, does not hold at high holdups or for dense packings of drops (*5*).

The identification of the corresponding coefficients leads to the following meanings of the parameters of the axial dispersion model: The number of dispersed phase transfer units, N_d, is equivalent to the number of times that the wake sheds a thermal capacitance equal to that of the drop. The number of continuous phase transfer units, N_c, is N_d times the usual heat capacitance ratio (Rr). The Peclet number for the continuous phase is the number of times the total input continuous phase stream is shed in the wake shedding zone relative to the total number of wakes and drops that can be formed in the two streams.

The wake model can therefore be regarded as a mechanistic theory of axial dispersion for the wake shedding zone. The above analogy can have a useful application. The axial dispersion model can be applied to tracer experiments in the wake shedding zone to obtain the Peclet number and the number of transfer units. These in turn can be used to obtain the wake volumes and the rate of wake shedding for comparison with the values obtained from the temperature profiles (*4*).

ACKNOWLEDGMENT

The authors are indebted to Dr. F. O. Mixon of Research Triangle Institute for the analogy to the axial dispersion model in the wake shedding zone.

NOTATION

F = dimensionless temperature of the continuous phase at the top of the wake shedding zone, defined by equation (33) of (*4*)
H = average holdup
L = length of the wake shedding zone, cm.
M = ratio of wake to drop volumes
m = volume of wake elements shed per volume of drop and unit length of drop travel, cm.$^{-1}$
N = number of transfer units

N_{Pe} = Peclet number
R = ratio of volumetric flow rates of dispersed to continuous phase
r = ratio of heat capacities per unit volume of dispersed to continuous phase
t = temperature, °C.
θ_a = temperature jump ratio, defined by equation (2) of (2)
x = dimensionless distance along wake shedding zone
z = distance along wake shedding zone, cm.

Subscripts

c = continuous phase
d = dispersed phase

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Prediction of Multicomponent Heats of Mixing for Simple Liquid Systems using Binary Data

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Frequently, the heat of mixing is required to complete an energy balance. Experimental values and reliable calculational techniques are available in many cases if the system is a binary, but for multicomponent systems both of these advantages disappear rapidly with increasing number of components. Because the latter situation is more likely to be of practical importance than the former, it is desirable to devise a method for calculating multicomponent heats of mixing from available data, either pure component or binary. While a relationship involving pure component data would be the more useful, correlation with binary data is a much less formidable task.

Various investigators have developed predictive equations for ternaries usually based upon the Redlich-Kister equation (*6*). The approach which seems to have gained widest acceptance is that of Scatchard (*7*), but the correlations proposed by Tsao and Smith (*9*), Knobeloch and Schwartz (*3*), and Schnaible, Van Ness, and Smith (*8*) also produce satisfactory results.

In the following development, an equation is derived based upon the lattice model for liquids rather than the Redlich-Kister equation. This expression is directly applicable to multicomponent mixtures and correlates available data with accuracy equivalent to the earlier methods.

LIQUID MODEL

Guggenheim (*1*) proposed his lattice model for strictly regular liquid solutions. Using this model, he developed an expression for the Helmholtz function of mixing for a binary solution. This expression can be extended to multicomponent solutions and becomes

$$-A^M/kT = \ln \left[\sum_{\substack{\text{all} \\ N_{ij}}} g(N_1, \ldots, N_m, N_{12}, \ldots, N_{m-1,m}) \cdot \exp\left(-\frac{1}{kT} \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} N_{ij} w_{ij} \right) \right] \quad (1)$$

where A^M is the Helmholtz function of mixing, k is Boltzmann's constant, T is the temperature, g is the number of ways the number of molecules of each component (N_i) can be arranged on the total number of lattice sites ($N = \sum_i N_i$) to give N_{ij} pairs of nearest neighbors, w_{ij} is the configuration energy of an $i - j$ pair, and m is the total number of components.

This equation may be simplified by assuming w_{ij} is zero (ideal solution), w_{ij} is so small that the lattice sites are occupied in a random manner (Bragg-Williams approximation), or w_{ij} is small enough that the various types of pairs do not interfere with each other (quasichemical approximation). The latter two assumptions will prove useful. By using the Bragg-Williams approximation, the most probable number of $i - j$ pairs is given by

$$N'^{2}_{ij} = \left(cN_1 - \sum_{p=1}^{m-1} \sum_{q=p+1}^{m} N_{pq} \right) \prod_{k=2}^{m} \left(cN_k - \sum_{p=k}^{m-1} \sum_{q=p+1}^{m} N_{pq} - \sum_{p=1}^{k-1} \sum_{q=p+1}^{k} N_{pq} \right) \quad (2)$$

which can be shown to be equivalent to

$$N'_{ij} = cN_iN_j/N = Ncx_icx_j \quad (3)$$

where c is the coordination number, N is the total number of molecules and x_i is the mole fraction of component i. If the quasichemical approximation is used, the most probable number of $i - j$ pairs is

$$N^{*2}_{ij} = \left[\left(cN_1 - \sum_{p=1}^{m-1} \sum_{q=p+1}^{m} N_{pq} \right) \prod_{k=2}^{m} \left(cN_k - \sum_{p=k}^{m-1} \sum_{q=p+1}^{m} N_{pq} - \sum_{p=1}^{k-1} \sum_{q=p+1}^{k} N_{pq} \right) \right] \cdot \exp\left(-2w_{ij}/kT\right) \approx N'^{2}_{ij} \exp\left(-2w_{ij}/kT\right) \quad (4)$$

HEAT OF MIXING

The heat of mixing can be calculated from the Helmholtz function (assuming negligible pressure-volume ef-

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